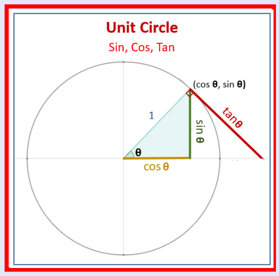
Math 241

Winter 2024

Lecture 2



Feb 19-8:47 AM

More Review

1) Solve $\frac{1}{2}x + 4 = \frac{2}{3}x - 5$

LCD = 6

$$\overset{3}{\cancel{6}}\cdot\frac{1}{\cancel{2}}x + 6\cdot 4 = \overset{2}{\cancel{6}}\cdot\frac{2}{\cancel{3}}x - 6\cdot 5$$

$$3x + 24 = 4x - 30$$

$$3x - 4x = -30 - 24$$

$$-x = -54$$

$$\boxed{x = 54}$$

Solution Set

$\{54\}$

Jan 3-8:06 AM

2) Solve, give final answer in interval notation

$$\frac{2}{5}x - \frac{1}{2} > \frac{1}{2}x - \frac{3}{5}$$

$(-\infty, 1)$

LCD $= 5 \cdot 2$

LCD $= 10$

$$\cancel{10}^{2} \cdot \frac{2}{\cancel{5}}x - \cancel{10}^{5} \cdot \frac{1}{\cancel{2}} > \cancel{10}^{5} \cdot \frac{1}{\cancel{2}}x - \cancel{10}^{2} \cdot \frac{3}{\cancel{5}}$$

$$4x - 5 > 5x - 6$$

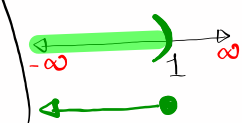
$$4x - 5x > -6 + 5$$

$$-x > -1$$

Now divide by -1

$$\frac{-x}{-1} < \frac{-1}{-1}$$

$\boxed{x < 1}$



S.B.N. $\{x \mid x < 1\}$

Such that

What is LCD for deno. 2, 4, 5, and 10?

$2 = 2$

$4 = 2 \cdot 2$

$5 = 5$

$10 = 2 \cdot 5$

LCD $= 2 \cdot 2 \cdot 5 = \boxed{20}$

Jan 3-8:09 AM

Solve by Factoring method:

$$x^2 - x - 12 = 0$$

$$(x - 4)(x + 3) = 0$$

Use Zero - Product Rule

$x - 4 = 0$ OR $x + 3 = 0$

$\boxed{x = 4}$ $\boxed{x = -3}$

Solution Set $\{-3, 4\}$

Solve by Factoring method:

$$3x^2 + 5x = 22$$

$\hookrightarrow$ RHS $= 0$ ✓

LHS must be factored ✓

$$3x^2 + 5x - 22 = 0$$

Product	-66	$-1, 66$
		$-2, 33$
		$-3, 22$
Sum	5	$-6, 11$

$$3x^2 - 6x + 11x - 22 = 0$$

$$3x(x - 2) + 11(x - 2) = 0$$

$$(x - 2)(3x + 11) = 0$$

By Z.P.R.

$x - 2 = 0$ OR $3x + 11 = 0$

$\boxed{x = 2}$ $\boxed{x = \frac{-11}{3}}$

Solution Set $\left\{-\frac{11}{3}, 2\right\}$

Jan 3-8:18 AM

Solve by Factoring:

$4x^2 + 9 = 12x$

$4x^2 + 9 - 12x = 0$

$4x^2 - 12x + 9 = 0 \rightarrow \underbrace{4x^2 - 6x}\ \underbrace{-6x + 9} = 0$

36

Product = 36

Sum = -12

-1, 36
-2, 18
-3, 12
4, 9
-6, -6

$2x(2x-3) - 3(2x-3) = 0$

$(2x-3)(2x-3) = 0$

By Z.P.R.

$2x - 3 = 0$ OR $2x-3=0$

$\boxed{x = \frac{3}{2}}$ Repeated Solution

Soln. Set $\left\{\frac{3}{2}\right\}$

Jan 3-8:28 AM

Solve $(2x+5)(3x-1) = 45$ by quadratic Formula

Foil, Simplify, RHS = 0

quadratic eqn

$ax^2 + bx + c = 0$

$6x^2 - 2x + 15x - 5 - 45 = 0$

$6x^2 + 13x - 50 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = 6 \quad b = 13 \quad c = -50$

$b^2 - 4ac = 13^2 - 4(6)(-50) = 1369$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-13 \pm \sqrt{1369}}{2(6)} = \frac{-13 \pm 37}{12}$

$x = \frac{-13+37}{12} = \frac{24}{12} = \boxed{2}$

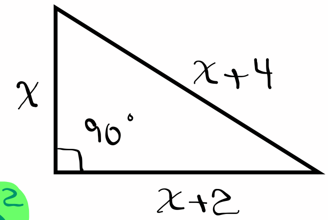
$x = \frac{-13-37}{12} = \frac{-50}{12} = \boxed{\frac{-25}{6}}$

Solution Set

$\left\{\frac{-25}{6}, 2\right\}$

Jan 3-8:37 AM

Find all three sides of a right triangle such that sides are 3 consecutive even integers.



By Pythagorean Thrm

$x^2 + (x+2)^2 = (x+4)^2$

$x^2 + \cancel{x^2} + 4x + 4 = \cancel{x^2} + 8x + 16$

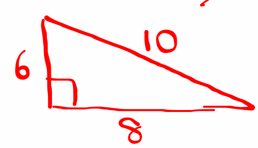
$x^2 + 4x + 4 - 8x - 16 = 0$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

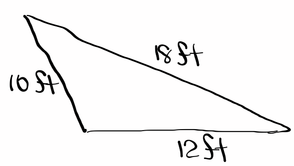
$x - 6 = 0 \qquad x + 2 = 0$

$x = 6 \qquad \cancel{x = -2}$



Jan 3-8:48 AM

Find the area of the triangle given below



Is this a right Triangle?

$a^2 + b^2 = c^2$

$10^2 + 12^2 \stackrel{?}{=} 18^2$

$100 + 144 \stackrel{?}{=} 324$

$244 \neq 324$

Not a right Triangle.

Use Heron's Formula

$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$

where $S = \dfrac{a+b+c}{2}$

$S = \dfrac{10+12+18}{2} = \dfrac{40}{2} = 20$

$\text{Area} = \sqrt{20(20-10)(20-12)(20-18)}$

$= \sqrt{20 \cdot 10 \cdot 8 \cdot 2}$

$= \sqrt{2 \cdot 10 \cdot 10 \cdot 16}$

$= \sqrt{16} \cdot \sqrt{100} \cdot \sqrt{2}$

$= 4 \cdot 10 \cdot \sqrt{2}$

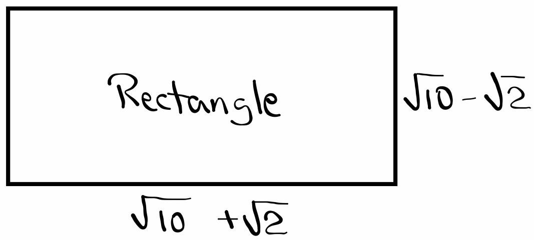
$= \boxed{40\sqrt{2}\ \text{ft}^2}$

$\text{Area} \approx 56.57\ \text{ft}^2$

$\approx \boxed{57\ \text{ft}^2}$

Jan 3-8:58 AM

Find area & Perimeter of the shape below



$A = LW$

$= \underbrace{(\sqrt{10}+\sqrt{2})(\sqrt{10}-\sqrt{2})}_{\text{conjugates}}$

$= (\sqrt{10})^2 - (\sqrt{2})^2$

$= 10 - 2 = \boxed{8}$

Recall $(A+B)(A-B) = A^2 - B^2$

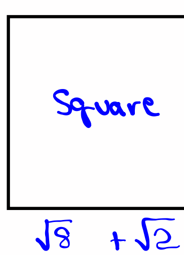
$P = 2L + 2W$

$= 2(\sqrt{10}+\sqrt{2}) + 2(\sqrt{10}-\sqrt{2}) = 2\sqrt{10} + 2\sqrt{2} + 2\sqrt{10} - 2\sqrt{2}$

$= \boxed{4\sqrt{10}}$

Jan 3-9:33 AM

Find area & Perimeter of the shape below



$A = S^2$

$= (\sqrt{8}+\sqrt{2})^2$

$= (\sqrt{8}+\sqrt{2})(\sqrt{8}+\sqrt{2})$

$= \sqrt{64} + \sqrt{16} + \sqrt{16} + \sqrt{4}$

$= 8 + 4 + 4 + 2$

$= \boxed{18}$

$P = 4S = 4(\sqrt{8}+\sqrt{2}) = 4\sqrt{8} + 4\sqrt{2}$

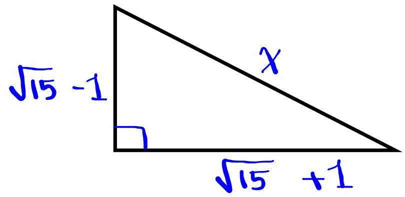
$= 4\cdot\sqrt{4}\sqrt{2} + 4\sqrt{2}$

$= 4\cdot 2\sqrt{2} + 4\sqrt{2}$

$= 8\sqrt{2} + 4\sqrt{2} = \boxed{12\sqrt{2}}$

Jan 3-9:37 AM

Find the hypotenuse of the right triangle below



Pythagorean Thrm

$a^2 + b^2 = c^2$

$(\sqrt{15}-1)^2 + (\sqrt{15}+1)^2 = x^2$

$(\sqrt{15}-1)(\sqrt{15}-1) + (\sqrt{15}+1)(\sqrt{15}+1) = x^2$

$\sqrt{225} \cancel{-\sqrt{15}} \cancel{-\sqrt{15}} +1 + \sqrt{225} \cancel{+\sqrt{15}} \cancel{+\sqrt{15}} +1 = x^2$

$15 + 1 + 15 + 1 = x^2 \rightarrow x^2 = 32$

$x = \sqrt{32}$

$= \sqrt{16}\sqrt{2}$

$\boxed{x = 4\sqrt{2}}$

Jan 3-9:42 AM

If $x^2 = K$, then $x = \pm\sqrt{K}$

Square-Root Method

ex: Solve $x^2 = 400$

$x = \pm\sqrt{400}$

$x = \pm 20$

$\rightarrow \{\pm 20\}$

Solve $x^2 - 50 = 0$

$\rightarrow \{\pm 5\sqrt{2}\}$

$x^2 = 50$

By Square-Root method

$x = \pm\sqrt{50} = \pm\sqrt{25}\sqrt{2} = \pm 5\sqrt{2}$

Jan 3-9:48 AM

Solve $(2x-1)^2 - 9 = 40$

$(2x-1)^2 = 49$

By Square-Root Method

$2x-1 = \pm\sqrt{49}$

$2x - 1 = \pm 7$

$2x = 1 \pm 7$

$x = \frac{1 \pm 7}{2}$

$x = \frac{1+7}{2} = \frac{8}{2} = \boxed{4}$

$x = \frac{1-7}{2} = \frac{-6}{2} = \boxed{-3}$

$\{-3, 4\}$

Jan 3-9:52 AM

Solving $x^2 + bx + C = 0$ by Completing the Square method.

$x^2 + 8x - 6 = 0$

$x^2 + 8x + 16 = 6 + 16$

$(x+4)^2 = 22$

take $\frac{1}{2}b$, then square it, and add to both sides.

Now by S.R.M.,

$x + 4 = \pm\sqrt{22}$

$\boxed{x = -4 \pm\sqrt{22}}$

Soln. Set.

$\{-4 \pm \sqrt{22}\}$

Jan 3-9:56 AM

Solve by Completing the square method:

$x^2 - 10x - 24 = 0$

Take $\frac{1}{2}$ b, Square it, add to both sides

Make sure $= 1$

$x^2 - 10x + 25 = 24 + 25$

$(x-5)^2 = 49$

Now S.R.M.

$x - 5 = \pm\sqrt{49}$

$x = 5 \pm 7$

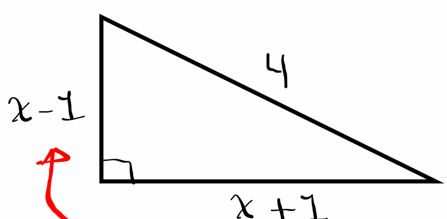
$x = 5 + 7$ $= 12$

$x = 5 - 7$ $= -2$

$\{-2, 12\}$

Jan 3-10:01 AM

Find x:



Use Pythagorean Thrm

$(x-1)^2 + (x+1)^2 = 4^2$

$x^2 \cancel{-2x} + 1 + x^2 \cancel{+2x} + 1 = 16$

$2x^2 + 2 = 16$

$x^2 + 1 = 8$

$x^2 = 7 \rightarrow x = \pm\sqrt{7}$

Can $x = -\sqrt{7}$? NO, $-\sqrt{7} - 1$ is a negative quantity.

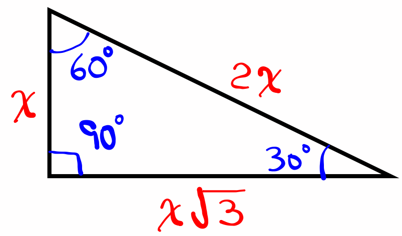
Side cannot be negative

$\{\sqrt{7}\}$

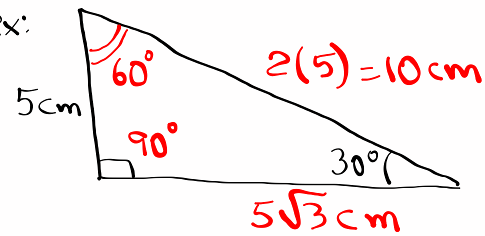
Jan 3-10:08 AM

Special Right Triangle

$30^\circ - 60^\circ - 90^\circ$



ex:



Verify

$5^2 + (5\sqrt{3})^2 = 10^2$

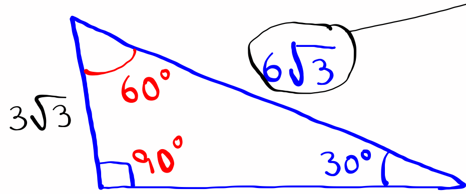
$25 + 25 \cdot 3 = 100$

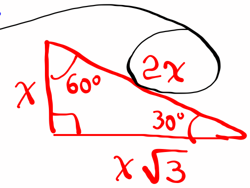
$25 + 75 = 100$

$100 = 100$ ✓

Jan 3-10:14 AM

Find missing sides & angle.



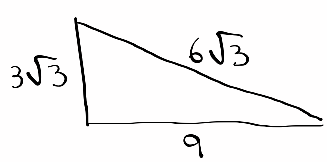


$3\sqrt{3} \cdot \sqrt{3}$

$= 3\sqrt{9} = 3 \cdot 3 = 9$

$2x = 6\sqrt{3}$

$\boxed{x = 3\sqrt{3}}$



Verify

$(3\sqrt{3})^2 + 9^2 = (6\sqrt{3})^2$

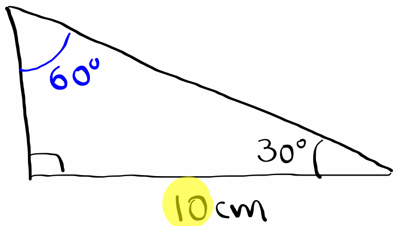
$9 \cdot 3 + 81 = 36 \cdot 3$

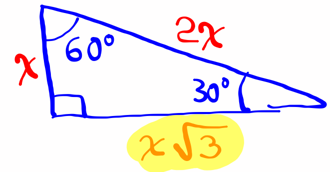
$27 + 81 = 108$

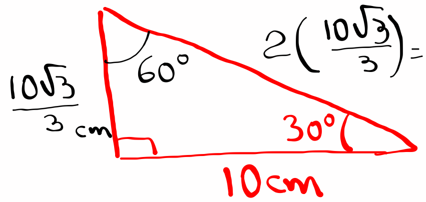
$108 = 108$ ✓

Jan 3-10:18 AM

Find missing sides & angle:







$$2\left(\frac{10\sqrt{3}}{3}\right) = \frac{20\sqrt{3}}{3}\text{ cm}$$

Method I:

$$x\sqrt{3} = 10$$

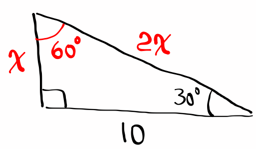
$$x = \frac{10}{\sqrt{3}} \cdot \boxed{\frac{\sqrt{3}}{\sqrt{3}}}$$

$$x = \frac{10\sqrt{3}}{\sqrt{9}}$$

$$\boxed{x = \frac{10\sqrt{3}}{3}}$$

Jan 3-10:23 AM

Method II



$$x^2 + 10^2 = (2x)^2$$

$$x^2 + 100 = 4x^2$$

$$4x^2 - x^2 = 100$$

$$3x^2 = 100$$

$$x^2 = \frac{100}{3}$$

$$x = \pm\sqrt{\frac{100}{3}}$$

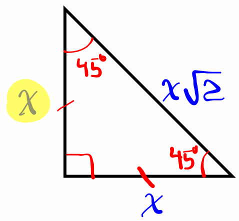
$$= \pm\frac{\sqrt{100}}{\sqrt{3}} = \frac{10}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

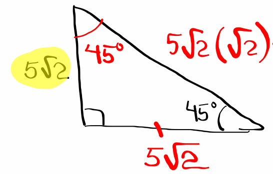
$$x = \pm\frac{10\sqrt{3}}{3}$$

$$\boxed{x = \frac{10\sqrt{3}}{3}}$$

Jan 3-10:29 AM

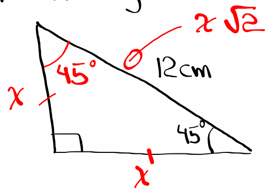
$45° - 45° - 90°$ Right Triangle





$5\sqrt{2}(\sqrt{2}) = 5\sqrt{4}$
$= 5 \cdot 2$
$= 10$

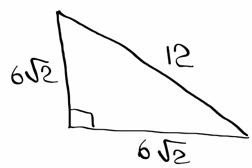
Find missing sides & angle



$x\sqrt{2} = 12$

$x = \frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{\sqrt{4}} = \frac{12\sqrt{2}}{2}$

$\boxed{x = 6\sqrt{2}}$



Verify

$a^2 + b^2 = c^2$

$(6\sqrt{2})^2 + (6\sqrt{2})^2 = 12^2$

$36 \cdot 2 + 36 \cdot 2 = 144$

$72 + 72 = 144$ ✓

Jan 3-10:33 AM

How to rationalize deno. when it has two terms in the form of

$\sqrt{a} \pm b$, $\sqrt{a} \pm \sqrt{b}$

Multiply top and bottom by the Conjugate of the deno.

ex: $\frac{2}{\sqrt{5} - \sqrt{3}} \cdot \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$

$= \frac{2(\sqrt{5} + \sqrt{3})}{\sqrt{25} + \cancel{\sqrt{15}} - \cancel{\sqrt{15}} - \sqrt{9}} = \frac{\cancel{2}(\sqrt{5} + \sqrt{3})}{\underbrace{5 - 3}_{\cancel{2}}}$

$= \boxed{\sqrt{5} + \sqrt{3}}$

Jan 3-11:06 AM

Rationalize the deno.:

$$\frac{\sqrt{5}}{\sqrt{5}+1}\cdot\frac{\sqrt{5}-1}{\sqrt{5}-1}=\frac{\sqrt{5}(\sqrt{5}-1)}{\sqrt{25}-\cancel{\sqrt{5}}+\cancel{\sqrt{5}}-1}$$

$$=\frac{\sqrt{25}-\sqrt{5}}{5-1}$$

$$=\boxed{\frac{5-\sqrt{5}}{4}}$$

Jan 3-11:10 AM

Rationalize the numerator:

$$\frac{\sqrt{7}-\sqrt{6}}{\sqrt{6}}\cdot\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}=\frac{\sqrt{49}+\cancel{\sqrt{42}}-\cancel{\sqrt{42}}-\sqrt{36}}{\sqrt{42}+\sqrt{36}}$$

$42 \to 42\cdot 1$

$21\cdot 2$

$7\cdot 6$

None are Perfect squares

$$=\frac{7-6}{\sqrt{42}+6}=\boxed{\frac{1}{\sqrt{42}+6}}$$

Jan 3-11:13 AM

Rationalize the deno.

$$\frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}+\sqrt{2}}\cdot\frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}-\sqrt{2}}=\frac{\sqrt{49}-\sqrt{14}-\sqrt{14}+\sqrt{4}}{\sqrt{49}-\cancel{\sqrt{14}}+\cancel{\sqrt{14}}-\sqrt{4}}$$

$$=\frac{7-2\sqrt{14}+2}{7-2}$$

$$-x-x=-2x$$

$$=\boxed{\frac{9-2\sqrt{14}}{5}}$$

$$\sqrt{A}+\sqrt{B}\neq\sqrt{A+B}$$

$$\sqrt{3}+\sqrt{2}\neq\sqrt{5}$$

Jan 3-11:18 AM

Rationalize the deno, them Simplify

$$\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}\cdot\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{\sqrt{9}-\cancel{\sqrt{6}}+\cancel{\sqrt{6}}-\sqrt{4}}=\frac{\sqrt{18}-\sqrt{12}}{3-2}$$

$$=\frac{\sqrt{9}\sqrt{2}-\sqrt{4}\sqrt{3}}{1}=\frac{3\sqrt{2}-2\sqrt{3}}{1}=\boxed{3\sqrt{2}-2\sqrt{3}}$$

Now rationalize the numerator:

$$\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}\cdot\frac{\sqrt{6}}{\sqrt{6}}=\frac{\sqrt{36}}{\sqrt{18}+\sqrt{12}}=\boxed{\frac{6}{3\sqrt{2}+2\sqrt{3}}}$$

Jan 3-11:27 AM

Solve

$3x^2 - 4 = 0$

$3x^2 = 4$

$x^2 = \frac{4}{3}$

Square-Root Method

$x = \pm\sqrt{\frac{4}{3}}$

$= \pm \frac{\sqrt{4}}{\sqrt{3}}$

$= \pm \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \boxed{\pm \frac{2\sqrt{3}}{3}}$

Soln Set

$\left\{ \pm \frac{2\sqrt{3}}{3} \right\} = \left\{ -\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3} \right\}$

Jan 3-11:36 AM

Draw a Circle Centered at (0,0) with radius 1.

In QI, find a point on the Circle that divides the curve by half.

$x^2 + x^2 = 1^2$

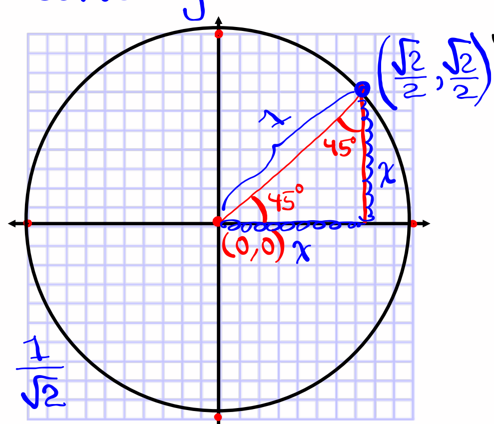
$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \sqrt{\frac{1}{2}}$

$= \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$



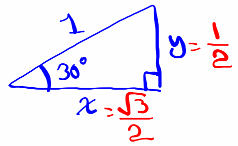
Unit Circle

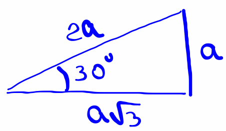
Center at (0,0) and radius 1

$x^2 + y^2 = 1$

Jan 3-11:40 AM

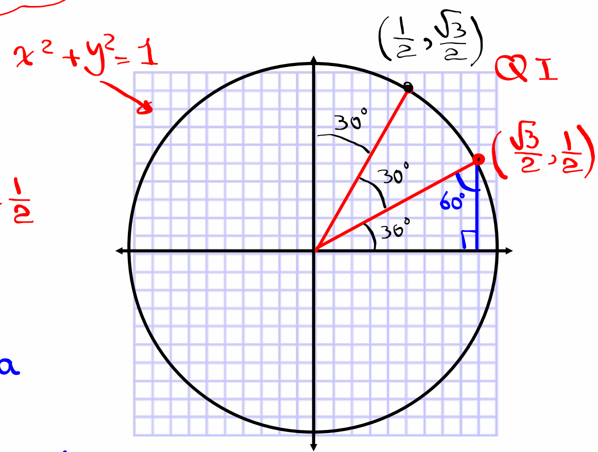
Consider a unit circle, find two points on the circle in QI that divides the curve into 3 equal parts



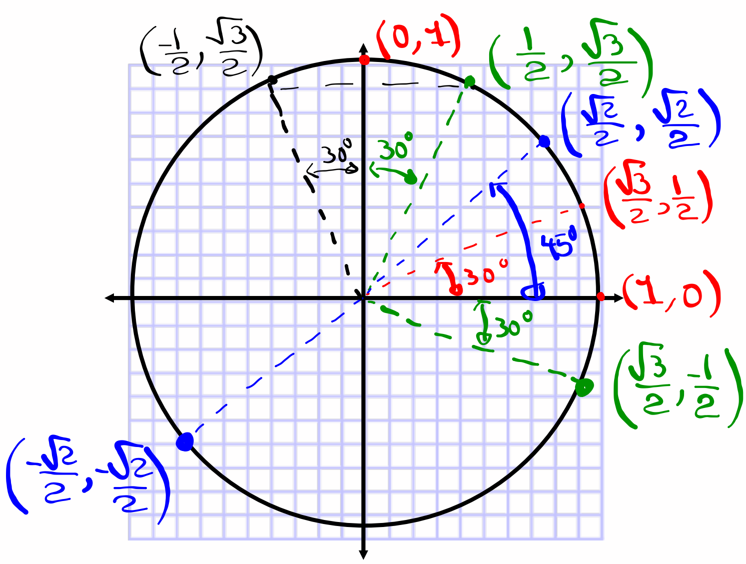


$2a = 1 \quad a = \frac{1}{2}$

$x = a\sqrt{3} = \frac{1}{2}\sqrt{3} = \frac{\sqrt{3}}{2}$



Jan 3-11:47 AM



Jan 3-11:58 AM

Class QZ 1

Find the area of a triangle with sides 8cm, 13 cm, and 11 cm. Box Your final Ans.

Round to a whole #

$S = \frac{8+13+11}{2} = \frac{32}{2} = 16$

$\text{Area} = \sqrt{16(16-8)(16-13)(16-11)} = \sqrt{16 \cdot 8 \cdot 3 \cdot 5}$

$= \sqrt{1920} \approx 43.818$

$\approx \boxed{44\ cm^2}$

Jan 3-12:03 PM